Midterm Revision T&F / MCQ

Determine whether the following statements are true or false:

1. A 5 × 6 matrix has six rows. False
2. A diagonal matrix is upper and lower triangular matrix at the same time. True
3. The matrix $B = A + A^T + A A^T$ is symmetric. True
4. If A and B are matrices of the same size, then $AB = BA$. false
5. If A and B are square matrices of same size, then $det(AB) \neq det(A) \cdot det(B)$. False
6. If A is a Square matrix with two proportional rows then $det(A) = 0$. True
7. If A and B are 2×2 matrices, then $AB = BA$. False
8. Trace of matrix is the product of the elements on the main diagonal. False
9. A single linear equation with two or more unknowns must always have infinitely many solutions. True
10. The matrix $B = A + A^T + A A^T$ is symmetric. True
11. If $Ax = 0$ has infinitely many solutions then $Ax = b$ will have no solution or infinitely many solutions but not a unique solution. True
12. A matrix is upper and lower triangular simultaneously if and only if it is a diagonal matrix. True
13. If A and B are square matrices of same size, then $det(A + B) \neq det(A) + det(B)$. True
14. The Number $(-1)^{i+j} M_{ij}$ is called the Cofactor of a_{ij}. True
15. If A is a Square matrix with two proportional rows then $det(A) \neq 0$. Flase

Midterm Revision T&F / MCQ

16. Vectors $(7,0,-2), (4,9,14)$ are orthogonal to each other.

True

17. $\mathbb{R}^2$ is a subspace in $\mathbb{R}^3$. False

18. All linearly independent set in a subspace W is a basis for W.

False

19. The transformation $\mathrm{T}: \mathbb{R}^2 \rightarrow \mathbb{R}^2, T(x, y) = 2x + 3y$ is a linear transformation. True

20. The column space of a 5×7 matrix is in $\mathbb{R}^5$. True

21. If A is $\mathrm{m} \times \mathrm{n}$ matrix then row space of A and column space of A have different dimension. False

22. If each component of a vector v in R^4 is *tripled*, then the norm of the vector is $3^4||v||$. False

23. Vectors $(a, 0,0), (0, b, 0)$ and $(0,0, c)$ are orthogonal to each other (where a, b and c are not zero). True

24. The initial point and terminal point of the vector $\overrightarrow{AB} = (2,1,-10)$ are $(3,-2,4)$ and $(5,-1,-6)$ respectively. True

25. The zero vector space $\{O\}$ has dimension0. True

26. Any linearly independent set in a subspace W is a basis for W. False

27. Let $\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3 \cdots \mathrm{v}_\mathrm{n}$ be vectors in the vector space $\mathbb{R}^\mathrm{n}$. Then the subset of all linear combination of these vectors is a subspace of $\mathbb{R}^\mathrm{n}$. True

28. The null space of A is the solution set of the equation $Ax = 0$. True

29. The column space of an $m \times n$ matrix is in $\mathbb{R}^m$. True

Midterm Revision T&F / MCQ

30. In the matrix transformation $T_A: R^n \rightarrow R^m$, $\forall$ vectors u and v: $T_A(u+v) = T_A(u) - T_A(v)$. False

(a) The norm of the vector $u = \frac{1}{\|w\|}.w$ is zero.

(a) False

(b) The vectors (3,7) and (3,7,0) are equivalent.

(b) False

(c) The set of vectors $\{(2,3,1),\ (-1,1,1),\ (4,6,7)\}$ is linearly independent.

(c) True

(d) The set $B = \{(1,2),\ (3,4)\}$ forms a basis of $\mathbb{R}^2$.

(d) True

(e) The dimension of a vector space is the number of elements in the largest linearly independent set in that vector space.

(e) True

(f) The dimension of row space and column space of a matrix is always same.

(f) True

Midterm Revision T&F / MCQ

(a) If (-2,3) and (4,1) are the initial and terminal points respectively then (-2,2) is the components of the vector.

(a) False

(b) If $\theta = 180°$, be the angle between two vectors then these vectors are orthogonal.

(b) False

(c) The set $\mathbb{R}^3$ is a subspace of $\mathbb{R}^4$.

(c) False

(d) The set $\{(1,2,1),(0,1,4),(6,12,6)\}$ of vectors in $\mathbb{R}^3$ is linearly dependent.

(d) True

(e) The basis of a vector space is not unique.

(e) True

(f) If A is a 3×3 matrix such that $|A| \neq 0$ then row vectors of A span $\mathbb{R}^3$.

(f) True

Midterm Revision T&F / MCQ

(a) The system of linear equations

$$2x - y = \frac{1}{2}$$
$$12x - 6y = 3$$

have a unique solution.

(a) False

(b) If A is 2×3 and B is 3×4 matrix, then $(AB)^T$ is the matrix of the size 4×2.

(b) True

(c) The matrix $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ is not invertible.

(c) False

(d) The matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$ is lower triangular but not upper triangular.

(d) False

(e) The determinant of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 4 \\ 3 & 0 & 3 \end{bmatrix}$ is 3.

(e) False

(f) The absolute values of minors and cofactors of the elements of a square matrix are identical.

(f) True

Midterm Revision T&F / MCQ

(a) Every system of linear equation is consistent.

(a) False

(b) The addition of two matrices is not possible only when there order differs.

(b) True

(c) The transpose of a lower triangular matrix is again lower triangular matrix.

(c) False

(d) If $A = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$, then $AB = \begin{bmatrix} 6 & 0 \\ 0 & -4 \end{bmatrix}$

(d) True

(e) The determinant of every non-singular matrix is zero.

(e) False

(f) The absolute values of minors and cofactors of the elements of a square matrix are not identical.

(f) False

For Each Question, Choose the Correct Answer from the Multiple-Choice List.

1. Determine whether the matrix below $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is in

   a)row echelon form   b) reduced row echelon form   c) both

Midterm Revision T&F / MCQ

2. If X is a 3×1 matrix and Y is a 1×2 matrix, then XY is

   a) 1×1 b) 3×2

   c) 2×3

3. The quantity $(\mathrm{B}^{-1}\mathrm{A}^{-1})^{\mathrm{T}}(\mathrm{B}^{\mathrm{T}}\mathrm{A}^{\mathrm{T}})^{2}(\mathrm{B}^{\mathrm{T}}\mathrm{A}^{\mathrm{T}})^{-1}$, is equal to

   a) $\mathrm{B}^{-1}\mathrm{A}^{-1}$ b) $\mathrm{B}^{\mathrm{T}}\mathrm{A}^{\mathrm{T}}$ c) I

4. The inverse of $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ is

   a) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

5. If A= $\begin{bmatrix} 3 & 1 & 2 \\ -1 & 4 & 5 \\ 0 & -3 & 6 \end{bmatrix}$, then the minor of a_{13} is:

   a) 3 b) -1 c) 0

6. If the determinant of $\mathrm{A} = {}^{1}/_{7}$,then $\det(\mathrm{A}^{-1})$:

   a) ${}^{1}/_{7}$ b) 7 c) $-{}^{1}/_{7}$

7. Determine whether the matrix below $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is in

   a) row echelon form b) reduced row echelon form c) both

8. If X is a 3×2 matrix and Y is a 2×1 matrix, then XY is

   a) 3×1 b) 2×2

c) 2×1

9. The quantity $(A^{-1}B^{-1})^T(A^TB^T)^2(A^TB^T)^{-1}$, is equal to
   a) A^TB^T b) I c) $A^{-1}B^{-1}$

$$(A^{-1}B^{-1})^T(A^TB^T)^2(A^TB^T)^{-1}$$
$$= ((BA)^{-1})^T((BA)^T)^2((BA)^T)^{-1}$$
$$= ((BA)^T)^{-1}\,(BA)^T\,(BA)^T((BA)^T)^{-1}$$
$$= I$$

10. The inverse of $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ is
   a) $\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$
   c) $\frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

11. If A= $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 2 & 3 \\ -1 & 5 & 6 \end{bmatrix}$,then the minor of $a_{32}$is:
   a)-14 b)20 c)8
   Minor of a_{32}=$(-1)^{3+2}\begin{vmatrix} 4 & 1 \\ -2 & 3 \end{vmatrix}$ = -1 · $(12 + 2) = -14.$

12. If the determinant of $A = {}^{11}/_{4}$,then $\det(A^{-1})$:
   a) ${}^{11}/_{4}$ b) ${}^{4}/_{11}$ c) ${}^{-11}/_{4}$

1. If $u = (5,1,4)$ and $v = (-1,0,2)$ are two vectors in $\mathbb{R}^3$. Then the cross product $u \times v$:
   a. $(-5,0,8)$
   b. $(4,2,6)$
   c. $(2,-14,1)$

d. $(0,0,0)$

2. Let $T_1(v_1, v_2) = (v_1 - v_2, v_1 + v_2)$ and $T_2(v_1, v_2) = (2v_2, 2v_1)$. The value of $T_1(T_2(v_1, v_2))$ is:
    a. $(2v_2 + 2v_1, 2v_1 - 2v_2)$
    b. $(2v_2 - 2v_1, 2v_1 + 2v_2)$
    c. $(2v_1 - 2v_2, 2v_1 + 2v_2)$
    d. $(2v_1 + 2v_1, 2v_1 - 2v_2)$

3. Let $S = \{v_1, v_2, v_3\}$ is a basis of V and $v_2 = 3v_1 - 5v_2$. Then the coordinate vector of V relative to S $((v)_s)$ is:

    a. $(3,5,0)$
    b. $(3,0,-5)$
    c. $(5,-3)$
    d. $(3,-5)$

4. A linear combination formed by the vectors $w_1 = (1,1,0)$, $w_2 = (0,1,-2)$ and $w_3 = (2,0,4)$ is:

    a. $w_3 = 4w_1 - 3w_2$
    b. $w_2 = w_1 + w_3$
    c. $w_3 = 2w_1 - 2w_2$
    d. $w_1 = -w_2 - w_3$

4. If u and v are two vectors in R^3(3-Space), then the vector $u \times v$ is perpendicular to

    a. u only
    b. v only
    c. both u and v

d. none of them.

5. If a, b and c are constants that are not all zero, then the equation $2ax + 2by + cz = 0$ represents
   e. a plane passing through (0,0,0)
   f. a plane passing through $(2a, 2b, c)$
   g. a line passing through (0,0,0)
   h. a line passing through $(2a, 2b, c)$

6. The set $V = \mathbb{R}^3$, together with the operation $r \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ r \end{bmatrix}$ and the addition is the standard operation on $\mathbb{R}^3$is not a vector space because:

   e. $u + v \neq v + u$
   f. $0 \notin V$
   g. $1 \times u \neq u$
   h. $u + (v + w) \neq (u + v) + w$

4. A linear combination formed by the vectors $x_1 = (1,0,-3)$, $x_2 = (1,-1,0)$ and $x_3 = (-2,3,-3)$ is:

   e. $x_3 = x_1 + x_2$
   f. $x_2 = 2x_1 + x_3$
   g. $x_3 = x_1 - 3x_2$
   h. $x_1 = x_2 - x_3$

2. Select one of the alternatives from the following questions as your answer. [6

(a) The matrix equation $AX = B$, where $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ corresponds to the system of linear equation

A.

$$\begin{aligned} 2x + 3y &= 0 \\ -x - 2y &= 1 \end{aligned}$$

B.

$$\begin{aligned} 2x + y &= 0 \\ 3x - 2y &= 1 \end{aligned}$$

C.

$$\begin{aligned} 2x - 2y &= 0 \\ 3x - y &= 1 \end{aligned}$$

D.

$$\begin{aligned} 2x - y &= 0 \\ 3x - 2y &= 1 \end{aligned}$$

(b) If If A, B and C are matrices of orders 3×4, 4×5 and 5×2 respectively; then the order of the matrix $(A.B).C$ is

A. 3×5

B. 3×4

C. 3×2

D. product is not possible.

(c) If $A = \begin{bmatrix} 2 & 2 \\ 5 & 6 \end{bmatrix}$, then A^{-1} is

A. $\frac{1}{2}\begin{bmatrix} 6 & -5 \\ -2 & 2 \end{bmatrix}$

B. $\frac{1}{2}\begin{bmatrix} 6 & 2 \\ -5 & -2 \end{bmatrix}$

C. $\begin{bmatrix} 3 & -1 \\ -\frac{5}{2} & 1 \end{bmatrix}$

D. inverse does not exists.

(d) The inverse of a upper triangular matrix is

A. upper triangular
B. lower triangular
C. does not exists
D. any matrix

(e) If $A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 1 & 2 \\ 3 & -1 & -1 \end{bmatrix}$ then the value of cofactor corresponding to the entry a_{32} is

A. -2
B. 2
C. 14
D. -14

(f) If A is a square matrix of order 3 with $\det(A) = 4$, then $\det(2A)$ is

A. 32
B. 16
C. 8
D. 4

Midterm Revision T&F / MCQ

2. Select one of the alternatives from the following questions as your answer. [6

(a) If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & -1 \end{bmatrix}$, then $((A^T)^T)^T =$

A. $(A^3)^T$

B. does not exists

C. $\begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & -1 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 4 & -1 \end{bmatrix}$

(b) If $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 10 \\ 4 & 7 \\ -3 & -4 \end{bmatrix}$, then $A + B^T =$

A. addition is not possible

B. $\begin{bmatrix} -2 & 5 & -1 \\ 12 & 8 & -7 \end{bmatrix}$

C. $\begin{bmatrix} -2 & 5 & -1 \\ 12 & 8 & -1 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 5 & -1 \\ 12 & 8 & -1 \end{bmatrix}$

(c) If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then matrix A is

A. upper triangular.

B. lower triangular.

C. diagonal matrix.

D. all of the above.

(d) The inverse of a lower triangular matrix is

A. upper triangular

B. lower triangular

C. does not exists

D. any matrix

(e) If $B = \begin{bmatrix} 3 & 2 & -1 \\ -1 & 8 & 7 \\ 4 & -3 & 1 \end{bmatrix}$ then the value of minor corresponding to the entry a_{22} is

A. 7

B. -7

C. 1

D. -1

(f) If $A = \begin{bmatrix} 2 & 1 \\ 4 & -2 \end{bmatrix}$, then adjoint of A is given by

A. $\begin{bmatrix} -2 & 1 \\ 4 & 2 \end{bmatrix}$

B. $\begin{bmatrix} -2 & 4 \\ 1 & 2 \end{bmatrix}$

C. $\begin{bmatrix} -2 & -1 \\ -4 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 2 & -1 \\ -4 & -2 \end{bmatrix}$

Midterm Revision T&F / MCQ

2. Select one of the alternatives from the following questions as your answer. [6]

(a) If $u = (1, 2, 0)$, $v = (4, 0, 6)$, then $d(u, v) =$

A. $\sqrt{48}$

B. 7

C. 48

D. 49

(b) If $u = (7, 3, -4, 5)$ and $v = (2, 1, -1, 0)$ then $u.v =$

A. $\sqrt{21}$

B. 13

C. 21

D. 12

(c) The set $A = \{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\}$ forms a basis of the vector space

A. M_{32}

B. M_{22}

C. M_{33}

D. M_{23}

(d) If $v = (2, 1, -2)$ and $\|kv\| = 12$, then the value of k

A. 4

B. $\frac{5}{2}$

C. $-\frac{5}{2}$

D. 3

(e) If $A_{n\times n}$ is a square matrix such that $|A| \neq 0$, then which of the following is/are correct

A. nullity of $A = 0$.

B. rank of $A = n$.

C. A is invertible.

D. all of the above.

(f) If A is $m \times n$ matrix, then

A. rank $(A) = n$

B. rank $(A) = m$

C. rank $(A) \leq \min(m, n)$

D. rank $(A) = m.n$

Midterm Revision T&F / MCQ

(b) If $u = (3, 1, 4, -6)$ and $v = (-3, -1, -4, 6)$ then the distance between u and v is

A. 0

B. 15

C. $\sqrt{248}$

D. None of the above

(c) Which of the following set of vectors in $\mathbb{R}^3$ is linearly independent?

A. $\{(1, 2, -4), (-8, 14, 6), (3, 4, -9), (1, 0, 0)\}$

B. $\{(1, 2, 5), (2, 5, 1), (1, 5, 2)\}$

C. $\{(1, 2, 3), (0, 0, 0), (3, 2, 1)\}$

D. $\{(3, 2, -4), (24, 16, -32)\}$

(d) The dimension of the vector space of 3×3 matrices of real numbers under the usual addition and scalar multiplication of matrices is

A. infinite

B. 9

C. 6

D. 27

(e) For which value of a and b the vector $w = (1, -3, 4)$ is a linear combination of $u = (2, 4, 0)$ and $v = (1, 4, -2)$ *i.e.* $w = au + bv$?

A. $a = 1, \quad b = -2$

B. $a = -3, \quad b = -2$

C. $a = -1, \quad b = -2$

D. None of the above

(f) If the rank of a 4×4 matrix is equal to 3, then

A. the matrix is invertible.

B. the dimension of the null space is 4.

C. the dimension of the null space is 3.

D. the dimension of the row space is 3.